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What Does It Mean to Reason Qualitatively?[1]

In the early years of computers, rapid progress in mathematical sciences resulted in a spectacular boost in popularity of sophisticated numerical equations and algorithms. Researchers and engineers focused on quantitative reasoning and representation methods, while qualitative methods were not even taken into account. Afterwards, scientists realized that in many cases human reasoning methods are far more efficient than any numerical algorithm. Tasks such as object recognition and classification, action planning, or reasoning about everyday activities could not be performed by computers as precisely as by human beings. Just like in many other fields of science, also in computer science researchers began working on imitating solutions known from nature. They wanted to model human-like reasoning methods in the hope of obtaining a higher efficiency level. The new, qualitative approach focuses exclusively on essential properties of a given task, whereas quantitative approach usually uses too precise data and is characterized by too complex numerical algorithms.

The qualitative methods have rapidly gained numerous followers among scientists working in different domains, such as physics, geography, robotics, mathematics, computer science, automatic control, etc. Qualitative approach enabled introducing new solutions that were unreachable in the quantitative approach, e.g., big-scale geographic systems (Egenhofer, Mark 1995), naive physical systems (Forbus 1988), and complex robotic reasoning systems (Suchan, Bhatt, forthcoming). It turned out that qualitative methods are the most appropriate to represent two main concepts of human everyday life, namely time and space. Computer scientists introduced many new spatial and spatio-temporal logics using qualitative approach (cf.

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Aiello, Pratt-Hartmann, van Benthem 2007, Muńoz-Velasco, Burrieza, Ojeda-Aciego 2013).

Due to its advantages, the qualitative approach has become very popular in various fields of science (cf. Bredeweg, Struss 2004). Nevertheless, the concept of qualitative approach is still improperly understood by many researchers and often mistaken for other concepts, such as nonnumeric, or relative approaches. This situation introduces unwanted problems into the discussion about qualitative methods, their applications, advantages, and disadvantages. In the article, we will explain what particular quantitative and qualitative concepts mean, what the quantitative and qualitative approaches are, and in what way qualitative approach can be obtained in various fields. In the end, we will discuss real applications of the qualitative approach in different domains.

1. STANDARD DISTINCTION: QUANTITATIVE–QUALITATIVE

Concepts of qualitative reasoning and qualitative approach, in general, are usually explained by referring to the distinction between qualitative and quantitative methods. The main idea is to describe what sorts of approaches may be called qualitative and, on the other hand, which methods should be treated as quantitative. We will explain the distinction by indicating features and examples of both approaches. In the following section we will describe the classical distinction between quantitative and qualitative approaches, which is similarly presented in numerous publications (e.g. Renz 2002, Gedig, Stiemer 2003, Forbus 2008).

The distinction is based on the difference between quantity and quality, which needs to be discussed first. A typical dictionary provides the following definition of these concepts (excerpts from American Heritage Dictionary; quoted also in Hernandez 1994):

quantity — 1. A specified or indefinite number or amount. An exact amount or number. 2. The measurable, countable, or comparable property or aspect of a thing.

quality — 1. The essential character of something; nature. 2. An inherent or distinguishing characteristic; property.

Dictionary definitions roughly indicate the meaning of quantity and quality. Quantity is an exact number or amount that describes specified, measurable aspect of an object, process, situation, etc. By contrast, quality is meant to be a characteristic property, an essence or even a nature of described object, process, etc. It follows from the definition that qualities are used in order to describe distinctive, most important features that may be expressed in some other manner than using exact numbers (but it is not forbidden to use exact numbers in qualitative description — we will pay special attention to this point in section 3).

It is worth stressing that in everyday life people generally make use of qualities while talking to each other, describing some facts, or even when they draw conclusions about the surrounding world. Quantities are mostly used by scientists and engineers, who communicate with one another in the numerical language of mathematics. Even so, it has been noted (Roy 2009) that they also make use of qualitative notions during their scientific work, especially when they undertake problems, interpret results of experiments, or describe complex processes.

Quantitative and qualitative approaches are based on the notions of quantity and quality but in order to understand them correctly, much more must be discussed. In following subsections, we will describe both approaches, their intuitive meanings and common features.

1.1. Quantitative approach

Quantitative approach is characterized by the involvement of quantities (i.e. specified, exact amounts or numbers) in describing properties and conducting reasoning. The quantitative methods are commonly used in classical mathematics, physics, automatics, etc. where equations with exact numbers are needed. An example of a quantitative equation for PD (proportional-derivative) controller (where the controller's output $u(t)$ depends on the current error $e(t)$ and the change of the current error $\frac{de(t)}{dt}$) is presented in (1).

(1) $$u(t) = k_p \cdot e(t) + k_d \cdot \frac{de(t)}{dt}$$

Since inputs of the equation (1), i.e. k_p, k_d, and $e(t)$ take exact values (exact real numbers), the value of the controller's output $u(t)$ is strictly determined by the calculation performance. It is worth noting that there are infinitely many values that $u(t)$ may take (it follows directly from the observation that there are infinitely many values that k_p, k_d, and $e(t)$ may take).

Quantitative variables usually take values from continuous or discrete and uniform scales of fixed values to provide a high precision of value representation. In a continuous scale of values, for every two values there is always a value between them, which may be described as in condition (2), where x, y, z are values from a continuous scale.

(2) $$\forall x \forall y [x < y \rightarrow \exists z (x < z < y)]$$

A scale of values is discrete and uniform whenever the distance between any two elements with no element between them is constant. This condition is formalized in (3), where x, x', y, y', z are values from discrete and uniform scale.

(3) $\forall x \forall y \forall x' \forall y' \{[(x < y) \wedge (x' < y') \wedge \neg\exists z(x < z < y) \wedge \neg\exists z(x' < z < y')] \rightarrow (|x - y| = |x' - y'|)\}$

The quantitative approach has the following, significant advantages:

• *Precise representation of values.* Since in many cases variables in the quantitative approach may take values from a continuous scale of values (e.g. real numbers), the exact and precise value of any input may be represented. On the other hand, when a discrete scale of values is used, there is usually a great number of values that variables may take and, as a result, values can still be represented precisely, albeit not in a strictly exact way. A real number approximation done by a computer or a calculator with many significant digits is a common example of such precise quantitative value representation which only uses a discrete scale of values.

• *Precise output of calculations*. Precise values of all inputs of a quantitative equation usually results in precise values of all outputs. This feature is highly desirable in most of mathematical and engineering systems where precise information is needed. Obtaining precise information is probably the most significant advantage of the quantitative approach.

Quantitative approach has also several disadvantages that make scientists search for other methods (Hernandez 1994). The most important weaknesses are:

• *Problem with uncertainty representation*. If the exact value of a variable is unknown (the information is uncertain), one exact value needs to be selected in order to calculate a quantitative equation. In such a case, uncertain information is treated as an exact one which in many applications is not allowed. Another method concerning uncertain information within the quantitative approach consists in taking into account all values that belong to the range of uncertainty for a given value. In other words, if it is uncertain whether a variable x equals 1, 2, or 3, then in further calculations each of those three cases needs to be considered and calculated. It is obvious that in the case of a wide range of uncertainty such a method leads to large computational complexity, which is usually unwanted.

• *Large computational complexity*. Many quantitative approaches face the problem of large computational complexity. One of the main reasons of this drawback is a fixed scale with a great number of values. There are usually far too many distinguished values which are unnecessary in a given application. Consider an example of HVAC (heating, ventilation, and air conditioning) system installed in a living room, where heating depends on the temperature inside the room. Since a change of temperature in the room is characterized by large inertia, there is no need to measure temperature with accuracy of 10 significant digits. A precise scale of values (with accuracy of 10 significant digits) would provide unnecessary increase in computational complexity.

• *Value falsification*. Even though the quantitative approach is said to be precise, in some cases, when the scale of values is fixed and discrete, information may be falsified. In such a scale, every value needs to be approximated to the nearest value from the fixed scale of values. Given a scale with values rounded to three decimal places, a car velocity that equals 64.3454 km/h would be approximated to 64.345 km/h, which probably does not make any difference in most cases. On the other hand, in some contexts there are important values that need to be represented exactly, e.g. the melting temperature of water. Let us consider a location (with low atmospheric pressure) where the melting point equals 0.63^0C. Approximation of this value, in the case of a fixed scale rounded to one decimal place, would equal 0.6^0C. It follows that a pipe with water at 0.64^0C would be recognized as a borderline case, which is actually not true ($0.64^0C > 0.63^0C$). It should be noted that this problem does not occur when the scale is not fixed.

Although the quantitative approach has some disadvantages, it is still the most commonly used method in science and engineering. Nevertheless, some researchers who are aware of drawbacks of the quantitative approach (mainly regarding computational complexity) have begun looking for an alternative approach.

1.2. Qualitative approach

The qualitative approach makes use of qualities, i.e. characteristic properties or essences. Its main aim is to provide suitable methods for human common-sense knowledge representation and reasoning (such methods have not been obtained by means of other approaches so far). It needs to be emphasized that the quantitative approach is in most cases unsuitable for representing human everyday reasoning methods. Forbus (2008: 361) stated that:

> People who have never heard of differential equations successfully reason about the common sense world of quantities, motion, space, and time. They do so often in circumstances offering little information, using the ability to characterize broad categories of outcomes to ascertain what might happen.

Therefore, common-sense knowledge is said to have qualitative nature. In everyday reasoning, people do not use the quantitative approach or quantitative equations:

> Many everyday physical phenomena, such as boiling, are not easily described by a single equation. And even when equations exist, people who know nothing about them can often reason fluently about the phenomena. So equations cannot be necessary for performing such reasoning (Forbus 1988: 240).

Additionally, due to numerous significant advantages of the qualitative approach, it is used not only for representing the common-sense, everyday reasoning mechanism but also for solving complex problems in engineering and science.

There is a variety of methods within qualitative approach, yet in order to characterize the whole approach, Forbus (2008) introduced three main principles:

• *Discretization*. Qualitative representation is equated with an abstraction of continuous properties. In order to represent continuous properties in a qualitative way, qualitative values (usually finitely many) and a mapping function from basic (continuous) values to qualitative values are introduced. A new scale of qualitative values is discrete but need not be fixed or uniform. As an example, consider the height of a table. Qualitative scale obtained by such a discretization may consist of three qualities, i.e. "small", "medium", "large". Note that, in general, one qualitative value represents many values from the initial continuous scale. However, it is not a rule, e.g. there may be a qualitative value that represents just one exact value from the initial continuous scale.

• *Relevance*. Discretization may be performed in many different ways by introducing various numbers of qualitative values or by changing a mapping function from initial values to qualities. The choice of discretization method should be relevant to the problem in question in order to imitate human-like representation methods that are used in everyday life. As an example, consider the quality "small" that refers to a soccer ball and the same quality "small" referring to a planet. Meaning of "small" depends on the object that is being described: a small soccer ball may have a diameter of 10-20 cm, while a small planet has much bigger dimensions, e.g. a diameter of 1,000-2,000 km. Furthermore, discretization method should be relevant to the precision that is required. Introducing a greater number of qualities usually results in a more precise representation.

• *Ambiguity*. We have presented a qualitative representation as an abstraction of continuous values. It follows that a qualitative representation usually contains less information than an exact quantitative representation. When we consider a small soccer ball, we do not know if it has a diameter of 13, 15, or 19 cm. Thus such a qualitative representation often results in an ambiguous situation in which no precise conclusions may be drawn from the obtained information. Ambiguity is therefore one of the common properties of the qualitative approach which may be seen either as a disadvantage or as a feature that corresponds to the human-like reasoning manner.

The qualitative approach has several significant characteristics that distinguish it from quantitative methods. The following features make the qualitative approach unique and suitable for numerous applications:

• *Qualitative scales of values are relevant*. There is no fixed scale of values, unlike in most quantitative methods. Instead, a scale of values may be freely changed by adding new values or by changing their meaning. Thanks to this feature, all problems mentioned in the previous section which may occur in a quantitative approach (problems with uncertainty representation, large computational complexity, and value falsification) can be avoided in the qualitative approach. In particular, uncertainty may be represented by means of ambiguous qualitative values, the computational complexity may be reduced by using a small number of qualitative values, and the value falsification problem may be omitted by adding suitable qualitative values for a precise representation of key values.

• *The qualitative approach enables representing the most important properties*. If the scale of qualitative values is relevant, qualitative values could be introduced in such a way that they only provide a crucial distinction between values. In other words, if in a given case the only important information is that the velocity is smaller than 100 km/h, there is no point in distinguishing objects with velocities from the interval [0 km/h, 100 km/h). Instead, one qualitative value "less than 100 km/h" may be associated with the [0 km/h, 100 km/h) interval. Such a situation may take place when a system for a speed camera with the speed limit 100 km/h is considered.

• *The qualitative approach is universal*. Since qualitative discretization causes ambiguity, and qualitative approach may be considered as abstractions of continuous values, the same qualitative approach may be suitable for a number of different situations. Hence, the same qualitative method may be used in various contexts.

Another important property of the qualitative approach is its cognitive adequacy, which, in general, means that the qualitative approach allows us to model human cognition — the representation and reasoning methods. Cognitive adequacy may be broken down into two concepts — conceptual adequacy and inferential adequacy (Renz 2002). Conceptual adequacy means that concepts used by humans are suitably represented. In other words, qualitative values occurring in the method correspond to concepts used by humans. Inferential adequacy is a property of approaches that have reasoning methods similar to the human reasoning mechanism. It means that computation methods, their complexity and precision, adequately represent human-like approach. Cognitive adequacy understood as a conceptual adequacy together with inferential adequacy is a desirable property in Artificial Intelligence systems that try to model human cognition. Unfortunately, it is extremely difficult to give a precise definition of cognitive adequacy and to determine whether a given approach may be treated as cognitively adequate. Cognitive adequacy of an approach cannot be deduced in theoretical research. Instead, it can be discovered during empirical psychological tests.

2. QUALITATIVE VALUES

In everyday life, people use finitely many linguistic values in order to describe continuous aspects of the surrounding world. This symbolic representation is by no means precise but somehow enables fluent communication and drawing correct conclusions. According to the distinction presented in the previous section, it can be stated that people make use of qualitative approach in most of the everyday situations. It is obvious that in some cases, e.g. in weighing vegetables at the greengrocer's, the quantitative approach is necessary. On the other hand, in everyday life, people usually make use of qualitative values while communicating with others, e.g. "John is tall, but Jane is way too short for her age". Symbolic values such as "tall" or "short" are purely qualitative. It would be difficult to give their appropriate quantitative definition. Try to ask a person who uses such symbolic values to specify their exact meaning and the quantitative values they correspond to. In most cases there will be no clear answer. Hence, qualitative values are commonly used even though no one can explain their precise meaning. What is important is that qualities enable us to distinguish essential properties of situations, objects, and processes. This is why they allow us to communicate and to infer in unexpectedly precise manner.

People can use a purely qualitative representation without knowing the exact import of symbolic values, whereas it is necessarily required to define a precise meaning of qualitative values if they are to be used in computer systems or other AI applications. It needs to be noted that in the qualitative approach the scale of values is not fixed. It follows that an informative content of different qualitative values may vary. Some qualities may correspond to single quantitative values, e.g. "the melting temperature", and other may designate wide range of exact values, e.g. "high temperature". There are numerous techniques to define the precise meaning of qualitative values. In what follows, we will describe four of the most commonly used methods. Note that each method adopts the quantitative approach in order to describe the exact import of qualitative values. Purely qualitative representation may be used by people but not by computer programs.

2.1. Singleton values

Singletons are specific qualities because they correspond to one precise value. As an example, consider the quality of "the melting temperature of water", which in defined conditions corresponds to one exact value — usually 0^0C (in standard conditions). Singletons (sets with exactly one element) are used when it is necessary to distinguish exact values, e.g. "absolute zero temperature", "sea level", etc. It is worth mentioning that the above values need not be represented by singletons in every system, because it is not always necessary to distinguish them. The examples are given just to demonstrate what singletons may be used for.

Another important point concerns similarity of singletons and quantitative values. Note that singletons have denotation similar to classical quantities: both of them correspond to one, exact and precise value. The natural question is whether singletons are identical to quantitative values. The answer is obviously in the negative. Singleton corresponds to one exact value because there is a need to distinguish such a value, whereas quantity may correspond to any value from a scale of values, whether there is a need to distinguish it or not. What is even more interesting is a comparison between quantitative scale of natural numbers and a qualitative scale with singletons that correspond to every single natural number. In such a case, both approaches use the same scale, but intentions differ. In the qualitative approach a scale of natural numbers is used because it is necessarily needed to distinguish every single natural number in a given situation, whereas in the quantitative approach no similar assumption is made. Hence, the explicit intention to distinguish particular values is significant when using singleton values.

2.2. Nonoverlapping subintervals

Among qualitative representation methods, the use of nonoverlapping subintervals is certainly the most commonly used. It is a classical way of representing qualitative values and is usually connected with the intuitive meaning of qualitative value. The method consists in dividing the continuous scale of values into a finite number of nonoverlapping subintervals. Each subinterval is denoted by its two boundary values (also called distinguishing values or landmark values), i.e. the beginning of the subinterval and its end. An example is presented in Figure 1, where two qualities are introduced, namely "short" and "tall". The quality "short" corresponds to the interval [0 cm, 160 cm] whereas "tall" corresponds to another interval — (160 cm, ∞).

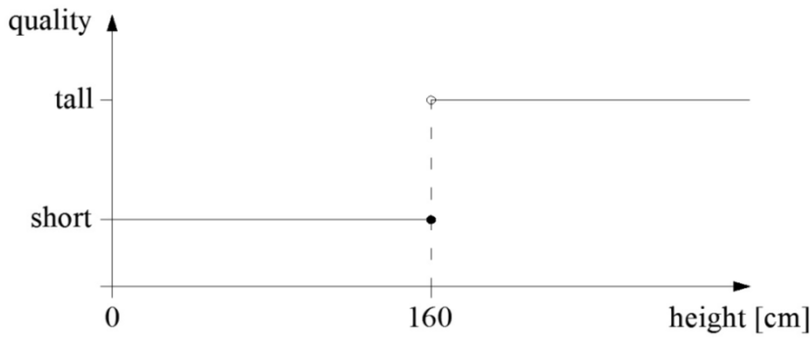

Figure 1. Qualities "short" and "tall" represented by means of two nonoverlapping subintervals.

The nonoverlapping-subintervals method is simple but has a significant feature desired in computer systems. Namely, every value from the continuous scale belongs to exactly one quality, i.e. to exactly one subinterval. The above property allows us to use algorithms of reasoning by exclusion or by cases, which are basic, powerful programming methods. Unfortunately, this feature leads to a significant disadvantage: the method is unable to express the vagueness of qualitative values. Symbolic values that are used in everyday life have fuzzy boundaries. For instance, consider the property of "being bald". It is impossible to define the exact boundary between "being bald" and "not being bald", because there is no exact number n of hairs such that a man with n hairs on his head is bald, whereas $n-1$ hairs would make him bald (the well-known Bald Man paradox of Eubulides of Miletus). In the following subsection we will present a popular method for representing qualities with vague boundaries.

2.3. Fuzzy sets

Another method of representing qualitative values consists in using fuzzy sets, i.e. each quality corresponds to a separate fuzzy set. A fuzzy set is a basic concept of the fuzzy logic introduced by Zadeh (1965). A fuzzy set A in a space of points X with a generic element $x \in X$ is characterized by a membership function $f_A(x)$ which maps x onto the interval [0, 1], as presented in (4).

(4) $A = \{(f_A(x), x): x \in X\}$

Intuitively, a fuzzy set is a more general notion than a set from the classical set theory. In the latter, each element either is or is not a member of a given set. By contrast, the notion of fuzzy set enables gradation of the relation of membership, i.e. each element is a member of a given fuzzy set with some grade of membership that belongs to the interval [0, 1], where 0 means that an element is not a member of a given fuzzy set, 1 means that it is a member of a given fuzzy set, and values from the interval (0, 1) represent fuzzy grades of membership, which cannot be expressed in the classical set theory. Fuzzy sets that correspond to the qualities "short" and "tall" are presented in Figure 2. In the given example, we can say that a person who is less than 130 cm tall is definitely short, whereas a person who is more than 190 cm tall is definitely tall. The boundary between "short" and "tall" is fuzzy and covers the interval [130 cm, 190 cm]. Note that it can be determined whether a person is rather short or rather tall even in the fuzzy boundary interval. Only a person who is 160 cm tall cannot be classified clearly to any qualitative value.

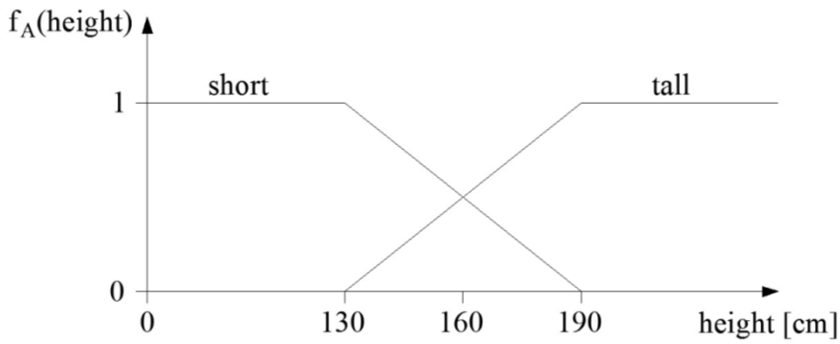

Figure 2. Qualities "short" and "tall" represented by means of two fuzzy sets.

Fuzzy sets may have different shape than those shown in Figure 2. The most commonly used shapes of fuzzy sets include triangular, trapezoidal, and Gaussian. A change of a fuzzy set shape affects the character of boundaries and allows us to determine the scope of uncertainty.

As we have already stated, the fuzzy-set method enables representing vague boundaries of qualitative values. Additionally, we can employ well-known fuzzy-logic theorems, operations, and their properties. On the other hand, if we use fuzzy sets as qualitative values, it is not enough to consider whether a particular point is a member of a given fuzzy set: we need to establish its grade of membership in the fuzzy set (which is equal to the membership function value). It follows that quantitative values (grades of membership) need to be stored and taken into account in further computations. Therefore, the fuzzy-sets method may be treated as a hybrid qualitative–quantitative approach.

We have stated that it is characteristic of qualitative approach that it lacks a fixed scale of values. In the case of fuzzy sets and nonoverlapping subintervals, definitions of qualitative values need not be fixed either. Instead, they may be chosen relevantly to the problem that is currently being solved.

2.4. Relations

The last method of representing qualities that we will describe differs from the previously presented ones, because when we want to describe relations between objects, values, or situations, there is no scale of values involved. In this case, qualities correspond to relations that occur between objects. This method is commonly used for representing spatial properties, e.g. in Region Connection Calculus (Li, Ying 2003), Allen's Interval Algebra (Allen 1983), and Egenhofer's method (Egenhofer, Franzosa 1991). Just like in previous methods, in order to define the meaning of distinguished qualities (relations between objects in this case), a precise formula needs to be introduced. As an example, let us consider a relation "*x* is close to *y*" where *x*, *y* are points in a one-dimensional space. Two objects may be treated as being close to each other if and only if the absolute distance between them is shorter than 10 cm, as defined in (5).

(5) $$x \text{ is close to } y \overset{Df.}{\Leftrightarrow} |x - y| < 10 \text{ cm}$$

Hence, qualitative relations also need to be defined by means of some quantitative equations. It should be clear now that every method of representing qualities that is to be used by a computer program needs to be precisely defined in a quantitative manner. One might argue that there could be other ways to define qualities, such as artificial neural networks, which do not use explicit equations for classification. Artificial neural networks, however, still make use of quantitative approach. The only difference is that they work like black box systems, i.e. we only know inputs and outputs, whereas we lack information about exact calculations that have been performed. Nevertheless, calculations performed by artificial neural networks are still quantitative, and the fact that we do not know their exact form does not make the approach non-quantitative.

Additionally, it should be stressed that different methods of qualitative representation can be combined. One of the most common representations is called "the domain of signs" and makes use of both singletons and nonoverlapping subintervals. There are three distinguished qualitative values in the domain of signs: "negative", "zero", and "positive". "Zero" corresponds to the singleton denoting the exact 0 value, whereas "negative" and "positive" are nonoverlapping intervals defined as follows:

(6) x is negative $\overset{Df.}{\Leftrightarrow} x \in (-\infty, 0)$

(7) x is positive $\overset{Df.}{\Leftrightarrow} x \in (0, \infty)$

3. ALTERNATIVE DISTINCTIONS

Although the distinction between quantitative and qualitative approaches is commonly used in the literature, it is often confused with some other similar distinctions. We have found the following commonly confused distinctions: fine–coarse, complete–incomplete, indefeasible–defeasible, absolute–relative, numeric–nonnumeric, and procedural–declarative. The confusion may stem from the fact that flagship examples used to demonstrate the idea of qualitative approach are usually coarse, incomplete, defeasible, relative, nonnumeric, and declarative, whereas quantitative approach examples are often fine, complete, indefeasible, absolute, numeric, and procedural. The above features may be connected, respectively, with the qualitative and quantitative approach but do not determine their essence. For example, it is not true that the quantitative approach is always fine and the qualitative is always coarse. In some cases the qualitative approach may also give fine results, and there are even cases in which the qualitative approach gives finer results than the quantitative approach. All depends on the situation and the methods used. Therefore, the quantitative–qualitative distinction does not determine other properties of the approach. In what follows, we will describe distinctions that are commonly confused with the quantitative–qualitative and explain why they cannot be equated with each other.

3.1. Fine–coarse

The distinction between fine and coarse refers to the level of precision of the results. Usually, the qualitative approach is considered to be coarser than the quantitative one. So far, we have only stated that results obtained in the qualitative approach are not exact but precise enough to solve a given task. However, it is not true that coarse results are a characteristic property of the qualitative approach. We have already indicated that, depending on the situation and methods used, the qualitative (or quantitative) approach may be both fine and coarse. As a counterexample to equating quantitative with fine and qualitative with coarse results, consider the already presented example. In a given place, the melting temperature of water equals 0.63^0C. In the quantitative approach, temperature is represented in a fixed scale with values rounded to one decimal place. It follows that the melting temperature in such a quantitative approach is denoted by 0.6^0C. In the case of the qualitative approach, where scale of values is not fixed, a singleton corresponding to the exact value of 0.63^0C is used to denote the melting point, and, additionally, two nonoverlapping intervals — "more than 0.63^0C" and "less than 0.63^0C" — are introduced. Hence, in the presented example the qualitative approach may produce finer results in terms of determining whether the temperature of water is above or below the melting point. For instance, in the quantitative approach, water at the temperature of 0.64^0C would be inaccurately classified as having the temperature of the melting point. On the other hand, in the qualitative approach the correct conclusion — that it is above the melting temperature — would be drawn.

3.2. Complete–incomplete

Another distinction that needs to be clarified is that between complete and incomplete reasoning methods. A system is (logically) complete when all of its tautologies are theorems. In other words, anything that is valid with respect to the system's semantics can be actually proven is this system. A more formal definition of completeness is presented in (8).

(8) Logic L is complete *iff* for every formula φ, if $L \models \varphi$, then $L \vdash \varphi$.

Obviously, there is no rule stating that quantitative systems are complete and qualitative are not. All depends on the structure of the system rather than on the choice between quantitative and qualitative approaches. In fact, completeness is a desired property in every logical system regardless of whether it is quantitative or qualitative.

3.3. Indefeasible–defeasible

A system is called defeasible when its reasoning methods are defeasible, i.e. conclusions drawn by the system not always follow solely from given premises. In other words, given true premises, one cannot be certain that conclusions are always true. Defeasible reasoning is a kind of non-demonstrative reasoning, where no complete demonstration of a claim is provided. By contrast, indefeasible reasoning may be identified with deductive reasoning, where true premises necessarily lead to true conclusions. In some cases, defeasible reasoning may be associated with qualitative probabilistic reasoning, but again, it is not a rule. The indefeasible–defeasible distinction has nothing to do with the quantitative–qualitative distinction: being indefeasible or defeasible is a result of inference rules incorporated into the system and not of the choice between the quantitative or the qualitative approach.

3.4. Absolute–relative

The absolute–relative distinction corresponds to the comparison method used in value representation. In the absolute approach every property is described according to an absolute scale of values which is usually fixed and given a priori, whereas if we use the relative approach, properties are described in terms of some relative values. The scale is usually not fixed and need not be known a priori. In some cases of the relative approach there is a relative scale of values that depends on some important, selected values, and in other cases there is no scale of values — instead, only relations between given properties are taken into account. An example of the relative approach of the first sort is a position of a pirate defined with respect to the treasure position. An example of the second sort is a system in which, in order to represent height of students, only relations: "taller than", "shorter than", "equal to" are used, and no scale of values occurs.

In many systems and examples presented in the literature, quantitative methods tend to be absolute, whereas qualitative ones are usually relative. In fact, it is natural to think of quantitative methods as absolute and of qualitative as relative, but at the same time the quantitative–qualitative distinction should not be confused with the absolute–relative one. Note that the absolute–relative division only distinguishes the form of scale of values that is used and does not decide whether the approach deals with quantities or qualities. In the following example a relative scale of values is used, but the approach is not qualitative. Consider a situation in which there is a buried treasure, and the only information important to the pirate is whether he is north or south of the treasure. The scale of values that is used is a three-dimensional and precise Cartesian coordinate system with real numbers for representing the pirate's position relatively to the treasure. The presented system is relative (it uses a scale of values relative to the treasure's position) but should not be regarded as qualitative, because it is not the case that only important and essential information is taken into account. Information that is being considered is far too precise and unnecessary in this situation. The example demonstrates that quantitative–qualitative and absolute–relative are two separate distinctions that should not be confused.

3.5. Numeric–nonnumeric

The numeric–nonnumeric distinction is probably most frequently confused with the quantitative–qualitative one, so the difference between them needs to be explained precisely. Stating that a method is numeric means that it uses some numerical analysis in its computations, where numerical analyses are, for instance, discretization, interpolation, extrapolation, regression, optimization, integration, and differential equations. The main condition is that numbers (numerical approximations) are used to represent properties of interest. Nonnumeric approach simply does not make use of numerical analysis.

In reference to the quantitative and qualitative approaches, we have stated that the quantitative approach makes use of quantities, i.e. exact amounts or numbers, whereas the qualitative approach uses qualities, i.e. characteristic properties, in order to represent properties. Hence, in most cases numbers are used as quantities and symbolic (usually linguistic) values are used as qualities. However, there is no rule stating that an approach is quantitative if and only if it uses numbers and qualitative if and only if it uses nonnumeric values. As a counterexample, we will show a qualitative approach that is numeric (uses numbers for representing properties and enables further numerical analysis). Thereby, we will show an approach that uses numbers for representing qualities. Let us consider a system that represents results of horse races, i.e. it is responsible for maintaining and processing information about places taken by jockeys at the end of a given race. For such a system it is essential to distinguish which place was taken by a jockey. Assuming that there are ten jockeys in every race, qualitative values corresponding to race results are in the form of the following natural numbers: 1, 2, …, 10. Note that this is a qualitative approach because only essential distinction in a given situation is made (what is distinguished are jockeys' positions and ordering on these positions). On the other hand, it is obvious that we are talking about a numeric method, since ten natural numbers are used in order to denote jockeys' results, and we can assume that various numerical methods of analysis can be performed using such a representation. Hence, the same method of representation (natural numbers in this example) may be used either in quantitative or in the qualitative approach. The quantitative–qualitative distinction, therefore, is not determined by a representation method but by the intention lying behind the choice of the representation method.

3.6. Procedural–declarative

The last distinction to be introduced is between procedural and declarative approaches and is commonly used by programmers. In the procedural approach, the whole procedure, i.e. the exact sequence of actions to be performed, is defined. Procedural programming languages include C++, Pascal, and Java. By contrast, in the declarative approach only definitions or constraints are declared, and no explicit procedure to be performed is given. There are several declarative programming paradigms, but two most popular are constraint programming and logic programming. The flagship example of declarative programming language is Prolog.

The distinction between procedural and declarative approaches concerns the way in which the method is presented and how the method's mechanisms are defined. It has nothing to do with the quantitative–qualitative distinction. Furthermore, most methods can be presented in both ways, i.e. procedurally or declaratively. The choice of the method's declaration depends usually on some practical aspects, e.g. on how it is easier to present a given method or which approach is more natural in a given case.

4. APPLICATIONS OF THE QUALITATIVE APPROACH

As we have already mentioned, the qualitative approach has numerous advantages, e.g. reduction of computational complexity, uncertainty modelling, and introduction of universal solutions. Hence, qualitative reasoning is used in order to establish physical models (Forbus 1988), Geographic Information Systems (Egenhofer, Mark 1995), robot navigation systems (Wagner, Hubner 2004), and numerous practical applications such as those presented in (Shimomura et al. 1995) or (Wałęga 2013). In what follows, we will present several examples of qualitative approach applications.

The concepts of qualitative values and qualitative reasoning led to the establishment of qualitative physics — a new way of modelling physical phenomena. Instead of describing physics by means of quantitative equations, a more intuitive qualitative approach is proposed. As an example, consider the case presented in (Forbus 2008), i.e. three containers with water connected by pipes, as shown in Figure 3.

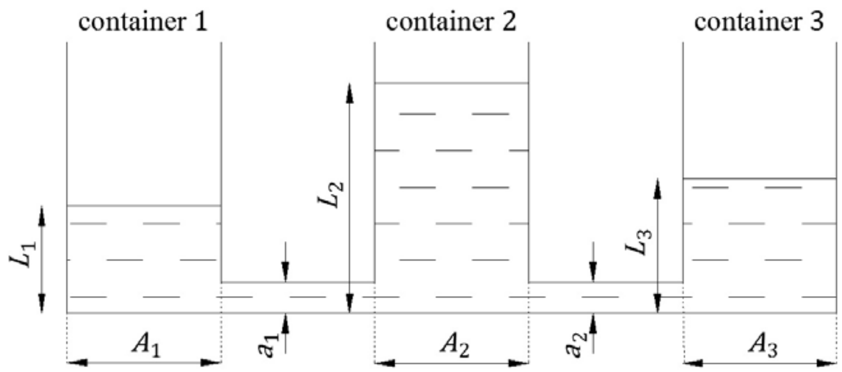

Figure 3. Three containers scenario, where L_1, L_2, L_3 denote levels of water in corresponding containers, A_1, A_2, A_3 denote their cross-sectional areas, and a_1, a_2 denote the cross-sectional areas of corresponding pipes.

Assuming that the level of water in container 2 is higher than in container 1 and in container 3, i.e. $L_2 > L_1 \wedge L_2 > L_3$, quantitative equations (9), (10), (11) can be introduced in order to model changes of water level in the containers.

(9) $$a_1\sqrt{2g(L_2 - L_1)} + a_2\sqrt{2g(L_2 - L_3)} = -A_2\frac{dL_2}{dt}$$

(10) $$a_1\sqrt{2g(L_2 - L_1)} = A_1\frac{dL_1}{dt}$$

(11) $$a_2\sqrt{2g(L_2 - L_3)} = A_3\frac{dL_3}{dt}$$

where:

a_1 — is the cross-sectional area of the pipe between the containers 1 and 2,
a_2 — is the cross-sectional area of the pipe between the containers 2 and 3,
A_1 — is the cross-sectional area of the container 1,
A_2 — is the cross-sectional area of the container 2,
A_3 — is the cross-sectional area of the container 3,
L_1 — is the level of water in the container 1,
L_2 — is the level of water in the container 2,
L_3 — is the level of water in the container 3,
g — is the gravitational acceleration.

If the containers were closed, the air pressure would have to be taken into account and equations would become even more complicated. Alternatively, the same physical process may be modelled qualitatively. Assuming that the initial situation is as previously, i.e. $L_2 > L_1 \wedge L_2 > L_3$, and that qualitative states describe only the flow direction (e.g. $1 \rightarrow 2$ means that water flows from container 1 to container 2, whereas the lack of an arrow would mean that there is no water flow between the containers 1 and 2), a process graph can be introduced, as shown in Figure 4.

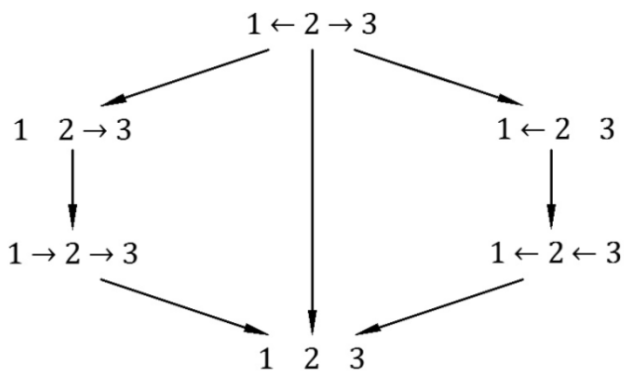

Figure 4. States graph for three containers scenario presented in Figure 3.

The initial situation is described by $1 \leftarrow 2 \rightarrow 3$, i.e. water flows from the container 2 into the containers 1 and 3. Big arrows in the diagram indicate state transitions that may occur. Note that it is not determined what the second state of the process is, it may be: $1\ 2 \rightarrow 3$, $1 \leftarrow 2\ 3$, or $1\ 2\ 3$. On the other hand, it is obvious that the last state is an equilibrium state, i.e. $1\ 2\ 3$, which will last unchanged. Such a qualitative-physics approach is more intuitive than the quantitative approach and usually involves lower computational complexity. There are also many interesting programming implementations of similar qualitative-physics processes (cf. Forbus 1988).

Another domain that makes use of qualitative reasoning is naive geography (Egenhofer, Mark 1995). It employs intuitive, human-like methods, just as qualitative physics does. Accordingly, naive geography could be called naive physics of geographic space. Its definition is as follows:

> Naive Geography is the body of knowledge that people have about the surrounding geographic world (Egenhofer, Mark 1995).

Therefore, naive geography should represent people's everyday methods of reasoning about geographic space and time. At the same time, space and time are two main common-sense concepts that are amenable to the qualitative approach (cf. Renz 2002). Hence, naive geography systems usually adopt the qualitative approach and, just like qualitative systems, may be sometimes inconsistent and contain errors. What distinguishes naive geography from qualitative physics is that qualitative physics focuses rather on the process mechanism, whereas in naive geography it is human interaction that plays the lead role. Moreover, naive geography introduces new spatio-temporal methods that are more accurate than the classical Cartesian coordinate space. It turned out that the intuitive pictorial way of representing geographical space is of different nature: it usually does not provide complete coordinates and carries other information, e.g. relative distance or size. Since naive geography is more intuitive than classical quantitative geography methods, it provides a greater number of valuable tools that may be used by a wide range of users without geographical education.

The qualitative approach is also used in automatic control, e.g. in order to introduce inference rules for a controller. Let us consider a classical PD controller (already introduced in section 1.1) which may be converted into a qualitative PD controller. The controller is a control loop feedback mechanism that attempts to minimize the error of a controlled value by adjusting the process control inputs. The PD controller uses two actions, namely proportional denoted by P and derivative denoted by D. Action P depends on the present error of controlled value, whereas D depends on the derivative of the present error of controlled value (in other words, action D makes a prediction of a future error, based on a current rate of error change). The quantitative PD algorithm was already presented in (1).

The PD controller may be introduced by adopting the qualitative approach. Firstly, the error value needs to be mapped onto qualitative values. Let us consider only two such qualitative values, namely negative, denoted by "*N*", and positive, denoted by

"*P*". Then, qualitative rules need to be introduced instead of the classical quantitative equation presented in (1). Assuming that N_e and P_e denote negative and positive values of error, while $N_{\Delta e}$ and $P_{\Delta e}$ denote negative and positive values of error derivative, qualitative control rules are introduced as presented in (12)-(15).

(12) $N_e \wedge N_{\Delta e} \rightarrow$ decrease input value

(13) $N_e \wedge P_{\Delta e} \rightarrow$ maintain input value

(14) $P_e \wedge N_{\Delta e} \rightarrow$ maintain input value

(15) $P_e \wedge P_{\Delta e} \rightarrow$ increase input value

Qualitative rules of control strategy are usually presented in the form of a table. Rules introduced in (12)-(15) are shown in Table 1

		e	
		N_e	P_e
Δe	$N_{\Delta e}$	**Dec**	**Man**
	$P_{\Delta e}$	**Man**	**Inc**

Table 1. The qualitative control rules of the PD controller, where Dec, Man, and Inc denote decreasing, maintaining, and increasing the input value.

The qualitative controller is based on intuitive rules, so its algorithm may be understood by a wide range of people. Using intuitive rules instead of quantitative equations is also very important when introducing control strategy which is based on experts' knowledge — usually obtained in the form of qualitative if-then rules. Moreover, qualitative approach usually leads to a decrease in calculation time and in more robust algorithms.

A fuzzy controller in many cases provides a precise solution. As an example, consider a ball-plate RT 123 experimental unit developed by GUNT Hamburg in collaboration with the Department of Automation and Information Technology at the Harz University of Applied Studies and Research. The RT 123 is a system with two actuators that are controlled by a fuzzy controller in order to move the ball to a specific position on the plate. The RT 123 behaviour is presented in Figure 5.

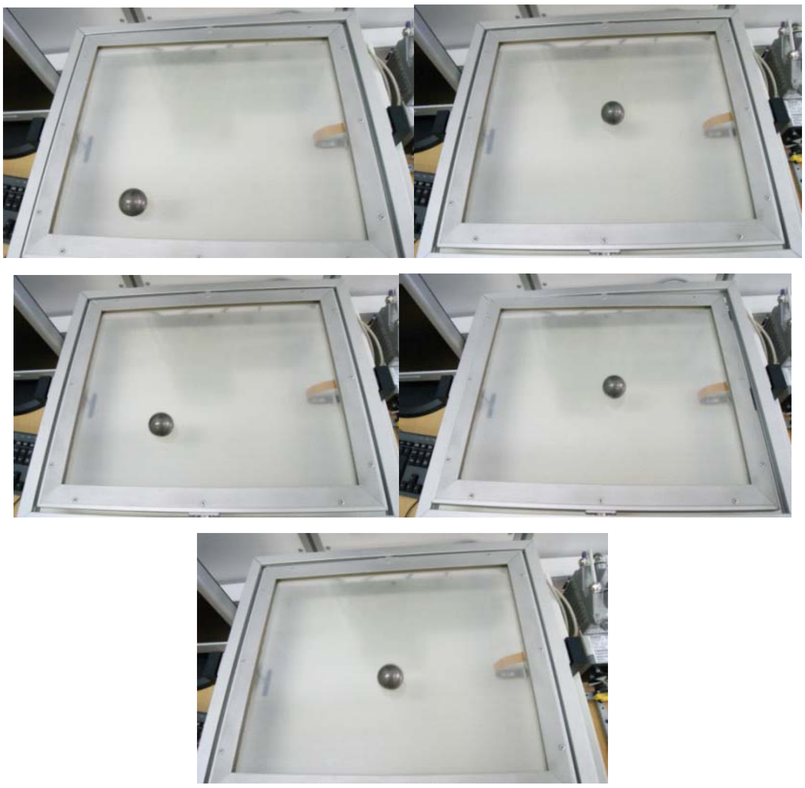

Figure 5. The sequence of photographs taken while RT 123 was moving the ball to the centre of the plate. The pictures were taken in the laboratory of Warsaw University of Technology, the faculty of Automatic Control and Robotics.

Fuzzy controllers are commonly used in many real applications, e.g. in robotics for movement control and collision avoidance.

The qualitative approach is also exploited in practical engineering systems, e.g. in photocopiers (for self-maintenance function) or in washing machines (for measuring the degree of soiling). The idea of a self-maintenance photocopier described in (Shimomura et al. 1995) provides state monitoring, fault detection, diagnosis, and two types of repair strategy — adjusting control parameters and reconfiguring the control structure. The qualitative approach is used in order to create a qualitative space map of input values. Three values measured by a toner density sensor, a photometer, and a surface electrometer are mapped onto the “fault” or “normal” qualitative values. Afterwards, the obtained qualitative values are used to identify faults and establish a repair strategy. The qualitative approach results in reduction of the size of the embedded software, increase in reasoning speed, and a much simpler reasoning algorithm than in the case of the classical quantitative approach.

5. CONCLUSIONS

Although a strict definition of qualitative reasoning has not been presented, we have (i) explained what types of inference are often confused with qualitative reasoning and (ii) provided examples of particular qualitative methods together with their applications. As a final point, we can suggest a sketch of a definition:

qualitative reasoning method — a human-like reasoning method that focuses exclusively on essential properties of a given task and is relevant to the problem to be solved.

The qualitative approach provides important advantages, such as intuitive, human-like representation of world's properties, low computational complexity, the ability to represent uncertainty and ambiguity. Thus it is commonly used in many domains, e.g. physics, geography, automatic control, robotics, etc. An increasing number of researchers working on qualitative methods and numerous successful applications of the qualitative approach give us solid grounds for claiming that the qualitative approach will play even more important role in the future. Therefore, a good understanding of the concept of qualitative approach and of the distinction between qualitative and quantitative approaches is needed. In particular, one has to draw a sharp distinction between the following, often confused concepts: qualitative, coarse, incomplete, defeasible, relative, nonnumeric, and declarative. Proper understanding of the concept of qualitative approach, of its features and methods of application, may increase the number of researchers working in the field.

We expect that in the near future numerous significant methods of qualitative reasoning will be put forward. In particular, we look forward to some progress in qualitative AI methods of knowledge representation and in methods of reasoning for robotic systems. Note that a strong trend of modelling and imitating human behaviour is currently observed in robotic systems. Thus the qualitative approach, which seems to represent human-like behaviour far better than the classical quantitative approach, is expected to be used in robotics more and more often.

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