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SOME PHILOSOPHICAL REMARKS ON THE CONCEPT OF STRUCTURE
FRAMING MICHAŁ HELLER'S PERSPECTIVE

Abstract

Perceiving objects in a structural or relational way in the ontology of physics and mathematics, as opposed to the classical way, shows how the concept of structure remains crucial for contemporary philosophy of physics and philosophy of science. In this paper, a particular emphasis is placed on certain philosophical concepts proposed by Michał Heller, concerning the context of the structural understanding of theories and the world. The first aim is to provide a general critical survey of the main assumptions of structural realism (SR). The second aim is to interpret Heller's philosophy of structure in accordance with the principal tenets of SR, illuminating certain criticisms of Heller's approach. Having analyzed Heller's approach, a question arises concerning the type of dependence on structuralism in the philosophy of physics and the philosophy of mathematics, in addition to certain metaphysical assumptions regarding the concept of structure. It is argued that Heller's SR conflates the adoption of mathematical structures in theories (the case of the realism–anti-realism debate) with the debate on mathematical explanations and the explanatory role of mathematical constraints.

Keywords: structure, structural realism, James Ladyman, Michał Heller, philosophy of science

Concerning the present dispute about the structural understanding of theories, certain philosophers have observed some metaphysical and mathematical consequences of the ongoing discussion. Perceiving objects in the structural or relational manner in the ontology of physics and mathematics, rather than in the classical way, indicates how the concept of structure continues to remain crucial for both the philosophy of physics and philosophy of science. This paper places a particular emphasis on certain philosophical concepts proposed by Michał Heller. First, a general picture of the ongoing discussion of Structural

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Realism (SR) will be delineated. Second, the issue of dependence on structuralism in the philosophy of physics and philosophy of mathematics will be discussed, alongside the problem of conceiving the world structure. Indeed, Heller proposed that scientific theories' mathematical structure crucially contributes to maintaining scientific theories' content in the face of changes, and according to Heller, the category theory potentially provides novel perspectives regarding the problem of mathematical structures. Accordingly, this paper has two aims. The first is to provide a general critical survey of the main assumptions present within SR. Second, this paper explains the advantages and disadvantages of Heller's perspectives on the philosophy of mathematics, as well as the concept of a mathematical world structure, within a structural framework.

1. FROM WORRALL TO LADYMAN

It is commonly held that the most powerful argument for scientific realism is the "no miracles argument," according to which science's success would be considered miraculous if scientific theories were not at least approximately true descriptions of the world (Putnam 1975: 73, Psillos 1999, Rodzeń 2005). Besides the "underdetermination argument," probably the strongest arguments against scientific realism are so-called historical objections, of which the best known is the pessimistic meta-induction (Alai 2017). SR, as introduced into contemporary philosophy of science by John Worrall, is concerned with the tension between these two powerful positions.

From the perspective of the historical development of science, particularly in relation to physics, one may observe the essential role of the concept of structure (Landry 2011). According to Worrall, continuous scientific development favors a realistic interpretation of physical theories. Worrall adopted the strategy of selective realism — that is to say, discerning between theories' stable and unstable components. He argued for an optimistic vision of scientific progress (continuity), given that at least some aspects of theories are retained, such as mathematical structures, whereas certain elements of theories (metaphysical assumptions) may be deemed erroneous and then rejected (theories change). Thus, according to numerous philosophers of science, SR appears to be a theory that saves the essence of the "no miracles argument" while being in line with the historical truth regarding the evolution of scientific theories (Worrall 1989).

After Worrall's paper, James Ladyman became a key participant in the debate on SR, as he proposed a further elaboration of Worrall's position. Specifically, he addressed a fundamental question concerning SR's nature — namely, whether it is metaphysics or epistemology. He observed that subscribing to SR does not mean that one must abandon traditional philosophical realism. As Ladyman clarified, SR must be regarded as metaphysical in contrast to being merely epistemically revisionistic (Ladyman 1998), given that its ontic version, taking structure to be primitive and ontologically subsistent, may diffuse the problems with both theory change and underdetermination.

2. STRUCTURAL REALISM AND ITS KEY FEATURES

The enduring debates regarding SR may be divided into two principal positions — namely, epistemic (ESR), according to which we have epistemic access only to the structures — and ontic (OSR), according to which we have epistemic access only to the structures, given that there is nothing other than structures (Esfeld, Lam 2011: 143-154, Ladyman 2014). ESR may be further subdivided into two types: there are unobservable individual objects although we cannot know them, or there may or may not be any such objects although we cannot know them in any case. Concerning OSR, the key thesis is eliminativism in the case of objects over which science draws a veil. Defenders of ESR may argue that OSR goes too far, because it does not remain agnostic about the existence of things that both sides believe to be epistemically inaccessible. Regardless, according to French and Ladyman (2011: 27-29), there are three compelling reasons for repudiating individual objects: (1) quantum mechanics' ontological implications (the received view in the philosophy of physics is that quantum particles are not individuals); (2) the fact that such objects belong to common-sense conceptions (the everyday macroscopic experience appears to be inadequate for fundamental metaphysics based on physical sciences); (3) the fact that the commitment to individual objects with intrinsic properties motivates haecceitism, which is seemingly incompatible with permutational invariance in quantum mechanics and with diffeomorphic invariance in general relativity. Nevertheless, besides these differences, OSR and ESR share at least three fundamental characteristics: (1) commitment to the claim that science is progressive and cumulative (the growth of structural knowledge surpasses empirical regularities); (2) the departure from standard referential semantics; (3) the position that scientific theories do not provide us with knowledge of the intrinsic nature of unobservable individual objects (Ladyman 2001: 55-69; 2017: 141-161).

French and Ladyman propose that OSR should not be perceived as the opposite of ontology, but rather as a reversal of traditional ontological tactics, wherein entities possess certain properties defined at the starting point. Regardless, the departure from perceiving objects in a classical manner in ontology may produce a fundamental challenge — namely, the existence of a relation network that is stipulated to be without things between which there are relations (Wolff 2012). Worrall's principal motive for introducing SR was merely to provide a realistic answer to pessimistic meta-induction, while French and Ladyman favored OSR due to two further problems. First, they touched upon the issue of identity and individuality of quantum particles and spacetime points along with the entanglement issue. Second, they undertook research into the scientific representation issue, particularly the role of models and idealization in physics. As previously mentioned, this reaffirms the metaphysical shift within SR since Worrall's paper (Ladyman, Ross 2007: 130-147).

Structure remains an essential concept in relation to OSR. As Ladyman stipulated, SR has a clear criterion — scientific theories' mathematical structure — for distinguishing between structure and nature, even if certain authors have strongly criticized the distinction between structure and content (Psillos 1999: 145-150). To more effectively comprehend the issue of structure, Ladyman proposed that one must transition from a syntactic to a semantic (model-theoretical) understanding of scientific theories. According to the semantic view, scientific theories are not mere sets of sentences and statements: they "are to be thought of as presenting structures or models that may be used to represent systems, rather than as partially-interpreted axiomatic systems" (Ladyman 1998: 416, Berenstain, Ladyman 2012). Consequently, a theory may be treated as a representation of a certain structure, pertaining to the world or a particular aspect of it. This explains why OSR places considerable emphasis on a theory's mathematical structure. Indeed, in contemporary physics, there are various formulations of the same theory, which in light of OSR would confirm that a theory's different formulations (for example in the case of Newtonian mechanics or quantum mechanics) are merely different representations of the same structure.[1] Subsequently, one may consider whether these different formulations are representations of the same ideal mathematical structure or of the physical (ontic) structure of the world. In this context, Ladyman is reliant on Weyl's view that if, after mutual transformation onto one another, different representations of the same theory re-

[1] In the case of Newtonian mechanics, one can consider its various alternative formal representations of the original Newtonian approach, associated with names such as Euler, Maupertuis, Lagrange, Hamilton, or Jacobi (Dieks 2019). Concerning quantum mechanics, among various formal approaches we can find the use of Hilbert's space (Schrödinger's, Heisenberg's, or Dirac's view), Feynman's integrals, C*-algebras, or statistical approaches by means of a density matrix (Heller 2014c: 165-172).

main invariant in relation to a certain group of transformations, then what remains invariant during these transformations corresponds to the state of things (Ladyman 1998: 418-421, Ladyman 2007: 145-147). Due to traditional realism, which assumes something more than merely structural characteristics of objects, these varied representations of the same structure are potentially a confirmation of the metaphysical engagement of structuralism. In fact, the current discussion within OSR concerning intrinsic properties and causation indicates how SR may avail itself of various metaphysical resources (French, Ladyman 2011: 33-40, Ladyman 2001: 69-74).

3. SOME PROBLEMS WITH STRUCTURAL REALISM

Structural Realism has been subject to various critiques, four of which will be presented in this section.

First, it is apparent that the primary example adopted to articulate SR's assumptions — the Fresnel–Maxwell case — is rather atypical in the history of science.[2] Worrall argues that it is unnecessary to have exact preservation of equations for SR to be defensible. Ladyman elaborates on this notion, proposing that OSR "is not claiming that the structure of current theories will be preserved simpliciter, but rather that the well-confirmed relations between the phenomena will be preserved in at least approximate form" (French, Ladyman 2011: 31-32). Regardless, certain philosophers have expressed concern that particular cases in the history of science may be pointed to where the equations have been transformed to such an extent that the structural continuity is questionable (Stanford 2003, Votsis 2011).

Second, it is apparent that without knowledge of particular objects, it is impractical to explain why certain properties and relations tend to have mutual cohesion. SR discusses realistic commitments only in some specific domains of the philosophy of physics (quantum mechanics and spacetime ontology). If this is the case, its range is particularly limited. By contrast, the fact that equations say something about the structure of the world may be better expressed in the context of the "no miracles argument," which is also theoreti-

[2] Fresnel's successful derivation of the reflection and refraction amplitudes for polarized light in the 19th century stands out as a significant contribution to the development of optical theories. Maxwell's theory produces equations for reflection and refraction that are formally equivalent to those of Fresnel's theory, as demonstrated by Lorentz in his doctoral thesis. This formal correspondence has been appealed to first by Poincaré and then by prominent advocates of SR (Worrall 1989) in defence of their structural realist position (Saatsi 2005).

cally engaged in statements concerning objects. Succinctly speaking, mathematical equations can confirm predictions only when they are interpreted theoretically and supported by auxiliary hypotheses (Psillos 1999: 145-149).

Third, OSR's foremost advantage is likely to be its limited type of realism, thereby permitting changes in theory pertaining to entities. Nevertheless, it is apparent that OSR simply accepts the current scientific understanding of quantum objects. We have sound premises to argue that these objects' scientific interpretations do change (there are different quantum theories). Accordingly, what is claimed by contemporary physics will be subject to change. Essentially, structural arguments based on current quantum mechanics do not offer a robust foundation for a decisive response to pessimistic induction, or for a definitive description at a quantum level. Rather, inductive and deductive formulations of the pessimistic induction appear to be fallacious in themselves (Mizrahi 2013).

Fourth, SR's main problem is that it fails to capture the difference between mathematical and abstract structure, on the one hand, and physical and concrete structure on the other (this idea is reconsidered in greater depth in Ladyman 2007: 220-238). This is the problem we will deal with in the remainder of this paper. As an ontological structuralist, Simon Saunders (2003) considers objects to be structures: the world has no ultimate constituents that are not themselves understood in structural terms. At the same time, he acknowledges that he is not committed to the belief that structures are merely mathematical or that his position is entirely neutral to Platonism. French and Ladyman emphasize that mathematics has an ineliminable role to play in theories, with OSR being able to be considered as friendly to a naturalized version of Platonism (Ladyman 2007: 157-158). If this is the case, while relinquishing common sense in metaphysics, OSR introduces another metaphysics to a certain extent — one that seems closely related to Platonist metaphysics of mathematical structures. Ladyman stresses that empirical sciences incorporate a verificationist — as opposed to Platonist — conception of reality. He explains that for a verificationist, within the Platonist vision (where there is a primacy of what is mathematically coherent), no perspective exists from which potential mathematical patterns' limitations may be discerned (Ladyman 2007: 234-235). Regardless, Ladyman acknowledges the particularly intriguing possibility that:

> the traditional gulf between Platonist realism about mathematics and naturalistic realism about physics will shrink or even vanish. The new wave of structuralism in the philosophy of mathematics, which has a number of supporting arguments in common with OSR adds substance to this speculation. (Ladyman 2007: 236-237)

Although Ladyman explicitly refuses to answer the question of what makes the structure physical and mathematical — claiming that "the 'world-structure' just is and exists independently of us and we represent it mathematico-physically via our theories" (2007: 158) — and is rather unsympathetic to Platonist realism, he admits the possibility that we may profitably explore structuralism in the philosophy of mathematics, as well as ask more explicitly what the meaning of the world as a mathematical structure is. Accordingly, the following two paragraphs are focused on how Michał Heller deals with these two concerns — namely, the structuralism in the philosophy of mathematics (section 5) and the world structure (section 6). First, however, I will briefly discuss the origins of Heller's SR (section 4).

4. ORIGINS OF HELLER'S STRUCTURALISM

To more effectively frame Heller's structuralist perspective, it is reasonable to consider the origins of his position more meticulously. Seemingly, Heller's deep interest in the concept of structure and SR was initiated by the doctoral thesis of his student, Krzysztof Turek. Turek's dissertation was titled *Strukturalne relacje między językiem, myśleniem a rzeczywistością* (*Structural Relations between Language, Thinking, and Reality*), which was the first doctorate defended at the Faculty of Philosophy of the Pontifical Academy of Theology in Kraków, in January 1983; it was prepared under Heller's supervision (Trombik 2019: 286). The defense of Turek's thesis was preceded by the publication of two articles in *Zagadnienia Filozoficzne w Nauce* (*Philosophical Problems in Science*) (Turek 1978, 1981, Krzanowski 2016). Turek's papers are mentioned in the premises of some of Heller's subsequent ideas. Here, two of them will be considered: the ontological interpretation of the concept of information, as well as the use of object-less categories.

Undoubtedly, Heller's SR involves numerous original concepts and makes significant departures from Turek's approach, as will be shown later. Nevertheless, it is important to clarify Turek's metaphysical view of information, in addition to its relevance for Heller's SR. Basically, information concepts may be categorized as either epistemological (namely, information as phenomena dependent on the existence of the conscious mind) or ontological (namely, information as fundamental elements of nature, irrespective of the existence of the mind). Turek perceives information in an ontological manner — that is to say, not reliant upon the existence of the mind but rather as a formative component of nature. His conceptualization of information is constructed using the Aristotelian concept of a form—matter composite. For Aristotle, whereas matter is the principle of potentiality and imposes individuality on every being, the form is the principle of its actuality and organizes matter into a substance. Notably, Turek argues that the concept of structure is contained in the concept of a form (Turek 1978: 32-36). He explains that there are, for example, forms reducible to structures that are investigated by the natural sciences and described by logic or mathematics; forms containing structures, albeit not reducible to structures such as the mind or irreducible living systems. Turek defines a formal ontology of information in two steps: first, he explicates the concept of information by using the notion of a form–matter composite; second, he provides an interpretation of information using the set-theoretical formalism. Despite the choice of Aristotelian metaphysics (potentially involving more terminological difficulties linked to the interpretation of Aristotle), the meaning of the terms "matter," "form," and "information" is highly restricted. Turek's proposal results in not being overloaded with metaphysical distinctions. In the case of applying the set theory to represent information as embodied structures, his proposal is significant yet incomplete. He observes that despite the similarities, the genus–individual structure (where the same essence exists in many things) and the structure of a set are not equivalent (Turek 1981: 73-76). Turek rightly notes that species is a concept pertaining to the real world, whereas a set is a mental construct: the membership in a genus is not merely defined in terms of the membership of its elements but by sharing a common essence.

Given the aforementioned discussion, where are we in terms of our question regarding Heller's SR, as well as the significant differences from Turek's approach? First, in various writings, Heller has emphasized that he is not particularly sympathetic to the Aristotelian philosophy (Heller 1992a: 72-82; 2007: 41-64). Accordingly, in what manner is Turek's approach connected with Heller's concerns with SR? It is not through the application of Aristotelian-like concepts, but, as I will argue, in a direct and indirect manner.

Regarding the direct manner, both for Turek and Heller, structures are systems with a given number of collections and relations between the elements of those collections. Consequently, both authors are interested in speaking about "nested structures," where a structure has elements that are other structures and where relations are of fundamental significance. Moreover, both Turek and Heller postulate an ontological interpretation of structures — namely, that reality-as-it-is is the web of "nested structures."

In an indirect manner, Turek's conceptualization of information and its connection to the set theory remains incomplete. Turek emphasizes the critical difference between a genus–individual structure and a set's structure.

This difference relates to the Axiom of Extensionality, which stipulates that two sets are equal if they have the same elements. Nevertheless, philosophical logic — axiomatized logic that may be applied to formalize humanities (for example metaphysical concepts) — is essentially a modal, intensional (rather than extensional) one (Galvan 1991, Huges, Creswell 1996). The internal difficulty of Turek's approach to formalizing the concept of structure stems from the fact that it is an open question as to whether the Z axiomatic set theory is an appropriate frame for Turek's formal ontology, deeply inspired by Aristotelian philosophical concepts. Seemingly, it is not. Indirectly, this explains why Heller has taken an alternative approach — the categorial one — to deal with the concept of structure.

5. STRUCTURALISM IN HELLER'S PHILOSOPHY OF MATHEMATICS

For many philosophers of mathematics, as well as for Heller himself, the structuralist interpretation of mathematics appears to be well-founded in both everyday mathematical practice and in metatheoretical investigations. Even so, a problem arises if one aims to articulate what is meant precisely by structuralism in the philosophy of mathematics. At this point, opinions start to diverge, and technical discussions replace a consensus (Reck, Price 2000). Heller (2006a: 160-161) proposes that we should at least distinguish between a weaker version of mathematical structuralism (mathematical objects treated as places in a structure, either devoid of an inner structure, or their structures being entirely determined by the structure in which they are substructures) and its stronger version (either the concept of structure does not require objects' existence at all or, if objects do exist, they are not cognizable). For Heller, the latter distinction has a strong metaphysical bearing, which transcends the foundations of mathematics. If we assume the stronger version, then the mathematical structures are not structures of anything. We could speak of mathematical objects understood as "empty places," devoid of any intrinsic properties in a structure. To decide between these two versions of mathematical structuralism, we should appeal to metaphysical or logical reasons. Moreover, concerning the nature of physics, Heller (2006a: 154-161; 2006b: 209-210) instead looks for the consequences of the structuralist assumption in mathematics in the weaker sense.

As Ladyman explained, "certainly, the structuralist faces a challenge in articulating her views to contemporary philosophers schooled in modern logic and set theory, which retains the classical framework of individual objects represented by variables subject to predication or membership respectively" (Ladyman 2007: 155). As far as the structure's varied technical aspects are concerned, two mathematical theories appear to be especially relevant — namely, set theory and category theory. Even so, such an "objectivist" perspective dominating the set theory, wherein the comprehension of the structure is reduced to the concept of sets and different relations between them, does not answer the question of how to think about sets themselves. Rather, Heller's attention is focused on the category theory. Indeed, the literature has identified different attempts to formulate the category theory without stipulating objects, where the primitives would be morphisms, compositions of morphisms, and the morphism of identity. Consequently, the concept of the object would be eliminated and such a theory's domain would consist solely of morphisms and relations between morphisms (without any elements). As a result, the whole of mathematics would be conceived as an overarching structure of structures. Although according to some scholars, such a theory potentially offers the prospect of identifying new mathematical foundations (Awodey 2004, MacLane 1997a, b), no consensus has been established regarding the identification of novel foundations for mathematics in these terms. However, Heller (2006b: 211-213) remains convinced that this theory's theoretical potential is sufficient to clarify the conceptual possibilities of perceiving the world as a network of relations and relations between relations, wherein the role of objects is entirely eliminated.

Heller (2016b) attempts to develop a kind of "philosophy of arrows" in two steps. The first step involves presenting an objectless (object-free) category theory in an axiomatic manner. The second step involves repeating a similar procedure, although at the level of the categories themselves, in order to formulate the category theory entirely in terms of functors. However, in this case, the identity functors replace the categories.

From the philosophical perspective, at least four implications are rather remarkable.

First, it has been established that the concept of category alone is unnecessary to develop the category theory ("categoryless, or category free, category theory").

Second, in the case of objectless category theory, objects do not acquire their individuality by definition, only formally through identity morphism. Essentially, we do not have to do this with the primitive thisness, only with the compositional thisness, given that in the case of objectless category theory, the context is provided by the morphisms that compose a certain identity (Heller 2016b: 452-453).

Third, in contrast with "set-theoretic ontology," wherein one attempts to define the individuality of the mathematical entities up to isomorphism, in category theory thisness is no longer determined by the arrow of the identity of a certain isolated object, but by the identities that correspond to appropriate morphism compositions (thisness up to isomorphism). Moreover, Heller analyses how the concept of isomorphism may be referred to categories via the concept of isomorphic functors, calling the isomorphism of such categories "thisness up to equivalence." As Heller suggests, this reveals formal properties that may be called "categorical ontology" (Heller 2014a: 446-448; 2016a: 263-264).

Fourth, from this conceptual analysis stems the conclusion that, even if the categories are complex entities (objects and their arrows), they may be reduced solely to the presence of morphisms. Consequently, we confront a different philosophy (ontology) of mathematics, which is no longer a philosophy of the elements (primitive thisness), but rather a philosophy of the arrows. It is no longer the set-theoretic ontology with the individuality of an element of a set given by its thisness independently of the relations to the environment; instead, it is a new ontology determined by the structure of the arrows, where we deal with certain structures' thisness that is reliant upon their context (compositional thisness, thisness up to isomorphism, thisness up to equivalence).

Based on the formulation of objectless and categoryless category theory, Heller illustrates two further significant philosophical implications. First, he clarifies that the concept of relation is excessively intertwined with commonsense ideas and the established theoretical approach. Consequently, such relational structuralism has a bottom-up character — that is to say, every relation must be between objects, thus ultimately arriving at some atomic elements. On the contrary, the categorial approach is seemingly a form of top-down structuralism, through which the same categories may be interpreted as the constitutive elements of categorial structuralism, which enables the building of a hierarchic universe of categories (Heller 2014a: 449-450; 2015: 186-187; 2016a: 260-261).

Second, even in the case of the old debate between the substantivalism and relationalism of spacetime structure (Ladyman 2007: 141-145, Heller 2016a: 258-259), Heller explains that category theory may contribute to developing some formal clarification. He proposes that the spacetime model should be based on Leibnizian philosophy, formalized as the Leib category within the category theory framework. This model is merely a form of thought experiment, which nevertheless corresponds surprisingly well to the key Leibnizian ideas of monad, pre-established harmony, and relational space (Heller 2015: 192-195). Therefore, for Heller, the application of the "philosophy of arrows" to physics may convey general concepts pertaining to the world's relational ontology in accordance with Leibniz's or Whitehead's philosophy.

Before moving on to discuss how Heller interprets structures in an ontological way, it is worth emphasizing that although a logic — such as the set-theoretic one — has the set-elementhood as a primitive, this cannot straightforwardly account for the Aristotelian type of metaphysics, as envisaged by Turek. By contrast, certain scholars remain persuaded that a category-theoretic logic permits this. For example, in his numerous studies, Gianfranco Basti argues for natural realism metaphysics. On the one hand, this is sensitive to the natural sciences; on the other, it builds on the category theory (Basti 2014). Providing a critical analysis of Basti's philosophy is not the aim here; rather, I want to emphasize that the categorial framework is more "operational" in dealing with the Aristotelian concept of form or structure. As mentioned previously, one must recall that the set-elementhood condition of "being an element of the universal class *V*" in any standard set theory is not a primitive in the category theory framework. Categorial objects need not fulfill a predicative set-elementhood primitive; rather, a reflexive morphism is necessary. In category theory, the set-elementhood condition is not presupposed, although it is justified (Lawvere 2005). On this basis, categorial language is in principle apt for justifying a formal ontology of dynamic processes, where the evolution of such processes generates the various domains of functions (predicates). Essentially, since natural kinds of things are constituted dynamically by the shared causal relations, which result from the numerous interacting environmental conditions, this constitutes the ontic (causal) foundation of natural kinds. This neither assumes universal properties or relations (the second-order logic) nor focuses solely on atomic elements (the first-order logic); instead, it provides a formal representation of the dynamic processes within a categorial framework. Indeed, if Aristotelian philosophy has been accurately comprehended, it is not a metaphysics of isolated substances such as "material points"; rather, it is a relational perspective of nature, wherein "subsistent thing" is not being isolated and independent of any relation but is being in itself in the sense of a reflexive relation. Subsequently, the fundamental Aristotelian distinction between substance and relation can be formulated as the distinction between reflexive (being-in-itself) and being-to-something-else relations (for example, causal or logical relations). Basti believes that these Aristotelian distinctions may be formulated in category-theory logic in virtue of the functorial dual equivalence between the categories of Stone coalgebras and Boolean algebras (Basti, Ferrari 2020). Such a project of formal ontology continues to be elaborated, but it already indicates that formal tools of category theory may provide the "missing link" for Turek's proposal.

As previously mentioned, Heller recently stated that the category theory framework deserves the utmost philosophical attention. Although Heller has been uninterested in interpreting the concept of structure likened to Aristotelian metaphysics, he has applied categorial tools, in the conviction that they help us to grasp the presence and role of structures in the realm of mathematics. Accordingly, I will now turn to the ontological aspects of his structural realism.

6. THE WORLD STRUCTURE

Heller attempts to explain in what sense the structuralist view on mathematics transfers into our knowledge of the physical world, given that "in opposition to instrumentalism, SR suggests that the mathematical structure of a theory does reflect the structure of the world (i.e., it reflects real relations between unobservables)" (Psillos 1999: 142). Heller's reasoning is adopted alongside subsequent main points: mathematics is a science of structures (which was discussed in the previous section); physics employs mathematical structures to model the world; therefore, the world as discovered by physicists consists of mathematical structures interpreted as world structures (Heller 2006a: 155). Although the majority of discussions in the philosophy of physics, as initiated by Worrall's paper, have been concerned with the realism and anti-realism debate, Heller is instead focused on the structuralist interpretation of physics (Heller 2006a: 161-162). From Heller's structuralist perspective, it follows that physical theories concern the structures of the world. In accordance with issues of SR discussed in previous sections, Heller proposes that "either structures, discovered by physical theories, are structures of something, i.e., there is a structured stuff, but this stuff is transparent for the method of physics (epistemic version), or such a stuff is absent (ontic version)" (2006a: 165). As he explains, to decide between these two versions, one should invoke some metaphysical or logical reasons. Heller's metaphysical reasons in the case of structuralist interpretation of physics are inspired by Quine's ideas, Weyl's perspective of objectivity and invariance, and the distinction between mathematics and Mathematics. The following paragraphs will consider these aspects.

First, in terms of Quine's ideas, Heller emphasizes the legitimacy of discussing ontology in relation to Quine's conceptualization — that is to say, an ontology that does not speak of what actually exists, but of what is presupposed by certain theories (models). As clarified by Heller, there is an intimate relationship between ontologies (in Quine's sense) of mathematical theories (structures) and ontologies of physical theories (models). The difference between them lies in the interpretation: the ontologies of physical theories are referred to the world, whereas the mathematical ones are not (Heller 2006b: 137-170; 2014b: 41-45). Even so, when Heller analyses the very method of physics, partially paralleling the precision of Quine's approach, he seeks to identify the "ontological commitments" of physics itself. Heller argues that broadly speaking, physics:

> presupposes three things: (A) a certain mathematical structure; (B) a part or the aspect of the world which a given mathematical structure is supposed to model; (C) "bridge rules" interpreting (A) in terms of (B); owing to these rules (A) serves as a mathematical model of (B). (Heller 2018: 15)

Heller's argument for structures' ontological interpretation may be reconstructed in the following informal way:

Premise 1. Every particular physical theory (or model) is an implementation of the scheme (A)—(B)—(C);

Premise 2. Making empirical predictions and experimentally testing them is done within this scheme;

Premise 3. The success of all particular physical theories rests on this scheme;

Premise 4. The success of physical theories without their reference to "what there is" would be inexplicable;

Conclusion. We are ontologically committed to thinking that "there are real structures."

Notably, it does not necessarily follow from assuming that we are committed to epistemic structures (i.e., various formal tools) that ontic structures (in other words, worldly structures) exist. Agreeing with Heller, we may state that without the aforementioned premises nothing can be done in physics; disagreeing with him, we may state that the physical method itself is rather blind to the ontological aspects of what it seeks to explain. The latter claim does not entail that we should treat the explanatory success of physics as miraculous, but it emphasizes that the ontological bearing of mathematical structures as at least approximately true descriptions of the world is debatable.

As for the second reason, concurring with Ladyman, Heller observes that no space exists for a straightforward explication that the structure presupposed by a given physical theory should be identified with the mathematical structure that this theory employs. Indeed, in physics, theories exist (for example, quantum mechanics) that admit more than one mathematical formulation (Heller 2014c: 165-172). Although this creates certain challenges for realism, Heller considers it to be a possible advantage for structuralism (Heller 2009: 99-100). As stated succinctly by Heller:

> And my claim is (like that of Ladyman) that precisely the collection of these "representation invariants" is what a given theory is about, what constitutes the ontology of this physical theory. . . . If we believe in the success of physics, we are entitled to claim that the structures of the successive physical theories (in the above sense) approximate, or at least are somehow related, to the Structure of the World. (Heller 2006a: 166-167)

For Heller, evidence that different mathematical structures of natural phenomena support the same experimental results is a strong argument for the claim that an unchanging reality underpins these varied epistemic structures (models). Thus, our different epistemic structures are just approximations of this ontic structure.

Further texts by Heller are significant for thoroughly comprehending his SR (Raine, Heller 1981, Heller 1992b, 2006c). They reconstruct space-time structures of various theories (beginning with Aristotelian dynamics and moving to Einstein's theory of relativity, then further still to the post-Einstein search for a unification of relativity and quanta) in terms of modern differential geometry, which he argues enables us to identify the logical connections between subsequent stages of science's evolutionary process. Heller is convinced that his reconstruction of space-time theories reflects at least some of the real features of the development of science, conveying a certain logic inherent in scientific progress. The most important conclusion from Heller's reconstruction is that each new explanatory structure is more general, and previous structures are a special case of the new one. As a consequence, the development of science is a process of real structure approximation by theoretical structures.[3] His approach seems to introduce some original insights into the polemic often referred to in the literature as a dispute concerning the rationality of science, or its evolution. Even so, it remains an open question as to whether such stylization of the history of physics offers a sufficiently strong argument for the ontological interpretation of structures. I will return to this question in the next section.

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[3] I thank one of the reviewers for emphasizing this point.

For Heller, in the context of the current debate on SR, an important argument for an ontological interpretation of structures, as they are understood in current physics, is their mathematical character. In this regard, a question arises concerning the extent to which structuralism in the philosophy of physics is dependent on structuralism in the philosophy of mathematics (Heller 2006b: 215-234). From Heller's perspective, there is a deep dependence. Indeed, on the basis of the spectacular development of the sciences underpinned by the mathematical method, Heller proposes a starting hypothesis — namely, attribution of a property to the world on the basis of which the world can be very effectively examined via the use of mathematics. For Heller, based on the success of methods of physics, it follows that the world is mathematical (Heller 2006b: 48-57). Essentially, he interprets mathematics in an ontic manner, as a property of the world itself.

Such an ontic view of mathematics makes it feasible for Heller to perform further analysis of the concept of "Mathematics" vs "mathematics." In Heller's view, the latter should be deemed a product of the human mind, something that is abstracted from the world. However, the explanatory effectiveness of our mathematics for describing the world is dependent on the existence of a deeper "Mathematics," which may be regarded as "a fabric of the reality." "Mathematics" is far more sophisticated than "mathematics" (various mathematical structures) as developed by humans. Applying our extant mathematical knowledge, it is only possible to approximate certain structures; that is to say, by knowing certain structures we may obtain knowledge about other structures. Comprehending structures' relationships in this regard is vital to grasp what "the structure of the world" means in this instance.

For Heller, mathematics is the study of relationships among certain entities, rather than the study of their nature. Further, from such a study, Heller infers that the world is ontologically structural, since "there are real structures." And yet Heller does not seem to offer any bridge to close the gap between the world and mathematics. Instead, he focuses solely on one horn of the dilemma, choosing to explore the world of purely mathematical entities (for example, in the category theory framework) and investigate the structural approach in physics from the mathematical position. He has not been clear about the logical or metaphysical criteria for deciding between the epistemic and ontic interpretation of structure within his SR. Therefore, there is an open question as to the implications of the structure of the world being mathematical.

7. SOME FURTHER CRITICISMS

We are finally in a position to propose the objections: first, it should be explicitly asked in the case of SR what it means that the structure of the world is mathematical, and second, clarification is required concerning mathematics' beneficial contribution to scientific theories or to SR.

Regarding the first objection, it is apparent that Heller, like Turek and Ladyman, assumes that "there is" the world structure: Ladyman claims that "mathematical structures are used for the representation of physical structure and relations, and this kind of representation is ineliminable and irreducible in science" (2007: 159). However, Heller attributes a property to the world on the basis of which the world can be very effectively examined via the mathematical method; this is a particular type of mathematical rationality, which Heller dubs "the mathematicality of the world" (Heller 2006b: 37-81). Even so, it seems insufficient to state that — since mathematics is ineliminable or indispensable to the creation of successful empirical predictions and to be able to preserve continuity in respective mathematical structures during theory shift — the structure is "the worldly ontological stuff."

Heller provides a particularly intriguing example of the structure evolution in the case of space-time theories, arguing that there is a logic of continuity in physical theories' reconstructed history and their commitment to structures. Even so, Heller believes that such a logic of evolution in space-time theories "is a consequence of the strong stylization of the history of physics" (Heller 1992b: 122). The fact that such a reconstruction is possible is itself a testament to the inherent logic in the evolution of scientific theories or models, as well as to our ability to make progress in our comprehension of epistemic structures and their interdependence. Nevertheless, such stylization, where certain criteria have been adapted to former theories in such a manner that the entire analytical procedure could succeed, does not directly show what kind of structure exists in the world. Even if the result of such an analysis is philosophically significant and suggests how "the structure of the world" should be construed from these logical indications, strictly speaking, it is only from the epistemic perspective that we show how structural evolution effectively fits in the logic of physical progress.

A proponent of SR may argue again that the "unreasonable effectiveness of mathematics" is the reason why we represent the physical world accurately (approximately) (Wigner 1995). Ultimately, a proponent of SR does not argue that there is a simple relation of inclusion (particular structures as a specific case of other structures); such a perspective would be too naive. Rather, analysis of the method of physics itself or the reconstructed history of scientific theories shows that we can comprehend relations between structures only when we possess a more general theory (structure), which is crucial for understanding the meaning of a structure's approximation in a given case. Mathematics' essential work in generating successful predictions may be subject to discussion, given the potential scenario where mathematics does not mirror a phenomenon's structure at the relevant scale. A general objection to SR may be formulated; namely, mathematics, mirroring the structure of the phenomenon in question, may have excessive structure — if the mathematics is more complex than necessary — or excessively scant structure, in circumstances where a structure was needed but ignored. This is not merely an abstract possibility; rather, it is a consequence of the fact that using mathematics can also indicate our epistemic ignorance. If the appropriate level of structural detail is not possessed, we are not justified in claiming that our structures will be retained during theoretical change.

Generally, there may be "false" models in scientific use. Such models assist us with envisaging the highly instrumentalist value of mathematics, since the mathematical dependencies of these models do not correctly specify the causal dependencies. Nevertheless, such models provide us with descriptive hypotheses concerning reality, even if they do not accurately reveal the hidden causes of the phenomena (Bokulich 2009).

The above observation concerning the use of false models as well as the adoption of mathematics in relation to the explanatory aims may further illuminate the challenge of ontic interpretation of mathematics and mathematical structures. First, since the mathematical dependencies of scientific theories do not necessarily specify the causal dependencies that produce the *explanandum*, we may remain uncertain about the extent to which our hypotheses precisely describe the world. Ultimately, caution should be shown when attempting to distinguish merely descriptive models from explanatory models.

Proponents of SR may respond here that we may possess the requisite experimental tests of our theoretical structure's adequacy for the *explanandum*. Indeed, advocates of SR may strengthen their argument by noting experimentally confirmed consequences of the theory not predicted by the theory's author. Again, this is the sort of indispensability argument for realism about mathematical entities (Berenstain 2017), committing us not only to the existence of abstract mathematical entities but to a metaphysical structural relation ("Mathematics").

The opponent may respond that certain models are merely useful tools through which observational data may be organized. However, we do not claim that from the fact that a certain model or hypothesis is non-explanatory it necessarily follows that it cannot play any descriptive or predictive role. As a formal tool, it may be applied more or less correctly to unravel the structure or organization of certain complex phenomena. Even if mathematical transformations between theories exist and mathematical structures are explanatory and predictively successful, a structural realist should ultimately provide further reasons to clarify why these representation invariants or epistemic commitments preserve claims concerning the structure of the world itself. Indeed, mathematical structures may be preserved across theory shifts, simply on the basis that certain areas of mathematics are more effectively understood than others (Pincock 2011: 73-78). Even if the concept of relation seems strongly interlinked with common-sense metaphysics, the increasingly abstract and mathematical ontology of modern physics does not have to accurately latch on to something genuine in the physical world. It is necessary to have additional conditions in order to be confident about the correspondence between ontologies of mathematical theories (structures) and ontologies of physical theories (models).

8. MATHEMATICAL EXPLANATIONS

If this criticism is justified, is there any room for considering the positive contribution that mathematics makes to scientific theories or to SR? Undoubtedly, mathematics plays a crucial epistemic role, given the understanding of mathematics that emerges from the debate in the philosophy of mathematics concerning mathematics' theoretical indispensability to science (Pincock 2007). This epistemic perspective implies comprehension of the fact that mathematics permits the specification of a restricted range of claims about the phenomenon in question. That is to say, mathematics permits scientists to be neutral on a wide range of questions concerning the physical system in question, as opposed to determining certain ontic commitments. Essentially, the abstraction of mathematical representation allows scientists to develop reasoning independently of the unknown properties of the phenomenon in question (Pincock 2007: 256-265). Nonetheless, it is significant to observe that mathematical explanation's utility is not simply grounded in a form of ignorance regarding the physical world's precise ontology. Because of its substantial generality, mathematical formalism can provide the straightforward unifying mathematical characteristic of sound explanatory power. Evidently, in numerous instances, structural (mathematical) explanations are not easily translatable into non-mathematical terms. However, as Ladyman and Heller appropriately observed, such explanations appear extremely significant in relation to the areas of physics that are highly detached from our common-sense experience.

Responding to Heller's SR proposal, I suggest that one should carefully distinguish between, on the one hand, the issue of mathematical structures in theories in the realism–anti-realism debate (namely, the representation of structures) and, on the other, the issue of mathematical explanations, where mathematical constraints are considered to have an explanatory role. The latter means that potentially distinctive mathematical explanations exist, which may be qualified as non-causal (due to not deriving their explanatory power from success in describing the world's causal relations) and which work by describing the framework inhabited by any possible causal relation. If such explanations are modal ones, through revealing that the *explanandum* is more necessary than ordinary causal laws are in virtue of their mathematical constraints, it does not follow that mathematical explanations exploit the world's network of relations (ontic structure) (Lange 2013). Even so, there remains a question as to why these mathematical modalities should be more necessary than ontic constraints.

Mathematical explanation's role within SR was characterized by certain philosophers, as previously mentioned, in terms of structural explanation. The question concerning scientific explanation has played a fundamental role in the philosophy of science since the early twentieth century. For three decades, causal accounts of scientific explanation were dominant (Campaner, Galavotti 2012), although a significant change has occurred since the mid-2000s with an expanding repertoire of non-causal explanatory strategies (Reutlinger 2017). The list of these strategies includes the structural explanation. Such explanation is provided by a mathematical model *M* about a physical fact on the basis of the existence of a relationship of representation between *M* and the physical *explanandum*. Structural explanation can establish a common ground for understanding the *explanandum* in question, independently of the numerous ontologies underpinning mathematical formulations of theories (Dorato, Felline 2011: 161-172).

However, structural explanation should not be conceived as a mere relabeling of the D-N model of scientific explanation. First, even if certain mathematical features of the model become essential and do represent a physical fact, it is the latter that is explained in mathematical terms, rather than the other way around. Second, the structural explanation appears to be a natural by-product of the semantic perspective of theories, in contrast to the D-N model, which is strictly linked to the syntactic view of theories (Dorato, Felline 2011: 172-174). The majority of structural explanations advocate the primacy of relational properties over intrinsic ones. Nevertheless, in the context of scientific structural explanation, a more natural concept would arguably be that entities' structural properties are explanatorily more significant than their intrinsic properties (Dorato, Felline 2011: 175). Crucially, this notion of capturing the explanatory practice is insufficient to argue for some form of OSR or ESR. Given that both acknowledge the existence of a physical web of relations, this recognition alone is sufficient to account for the structural explanation's explanatory character. The choice between different forms of ESR or OSR is ultimately based on arbitrarily chosen metaphysical or logical reasons. Consequently, the claim that the physical method itself commits us to certain ontological statements about the physical world structure cannot be regarded as a definitive argument for any of SR's known forms.

In recent decades, the majority of realists discussing the sense of continuity or correspondence between theories and reality have developed these issues in response to antirealist challenges. Among other realistic approaches, structural realists have also advocated their "recipe realism," proclaiming their epistemic creed: "all we know is structure." Recipe realism's core concept involves searching for a universally applicable realistic explanation of science's success (Kotowski 2018). Saatsi posited this notion as being severely flawed, due to the diversity of science, pluralism of realist explanations, alongside the general difficulty for the realist to pin down the precise content of his recipe. As a result, Saatsi proposes replacing it with "exemplar realism," conceived of as being "at the same time global, in its attitude, and local, in its action." Such realism should concentrate on the piecemeal evaluation of the various cases in order to make room for a natural sense in which we can be realists about the majority of science (Saatsi 2017).

Even if Heller declares that he is not directly concerned with the realism—anti-realism debate, since his concern is with the structuralist interpretation of physics, his effort to interpret the latter issue seems to be marked by a sort of the recipe approach — that is to say, searching for the global consequences of structuralist assumptions in mathematics. I consider Heller's formal reconstruction of the evolution of space-time theories, formulation of objectless and categoryless category theory, or Leib category, as philosophically significant and capable of illuminating problems within some exemplar models, as opposed to being able to clarify the relation between structuralism in the philosophy of physics and structuralism in the philosophy of mathematics.

CONCLUSION

Presently, the concept of structure remains crucial for both the philosophy of physics and the philosophy of science. However, as has been argued here, many philosophical questions arise concerning the correlation between metaphysical and epistemological engagement in dealing with the concept of structure. Certain structural realists have advocated some crucial theoretical points that certainly affect the current debate regarding realism: the concept of structure in physics and the philosophy of mathematics; the shift from set-theoretic logic to category theory logic; the issue of structural explanation.

However, representation via formal methods is the crucial factor, given that mathematical or logical language is very regularly the scientific technique for explicating target systems' complexity. Nevertheless, scientists are unable to work merely by deducing theories from observations, or vice versa. Both experimental knowledge and various formal tools are requisite yet insufficient to formulate sound scientific explanations and proper comprehension of the *explanandum*. Very broadly, the way in which sciences are working seems to be a move from observations to formulating problems that require resolution, then attempting to formulate explanations combined with experimental checking, followed by further refinements not only of the theoretical framework but the *explanandum* itself.

Not only do various forms of scientific practices, models, and explanatory aims play crucial roles in providing good explanations, but there are also numerous formal systems that potentially present different sorts of benefits to the experimental ones, as Heller correctly proposes. However, my analysis may have general implications for the problem of how to bridge the gap between structuralism in the philosophy of physics and structuralism in the philosophy of mathematics. Specifically, philosophers of science have opened up the black boxes of the scientific enterprise regarding the demands of various scientific disciplines. Simultaneously, mathematicians and logicians have developed a multiplicity of formal structures and theories. The plethora of positions is apparent on both sides, while we can freely choose what we want to work with. Perhaps such fragmentation of our theorizing has revealed the bitter fruit. If this is the case, then — more than ever — we require a sound combination of the history of science, philosophy of science, and logic, as a means of avoiding the too-successful enculturation of philosophers into the scientific mindset and of scientists into the philosophical milieu. This balance is challenging to attain, although it is necessary and worth pursuing.

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